

Computing Channel Slope from a DEM: A Review of Issues and Two Algorithms

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Abstract — The problem of computing channel slope from a DEM is distinct from the problem of computing topographic slope in other parts of the landscape. This problem is important in the context of spatially distributed hydrologic modeling that starts with a DEM. Here, the challenge of computing good estimates of channel slope is explained, and then two different algorithms are described and compared. One of these, referred to here as the "slope-from-area algorithm", was published in a previous paper while the other is new and is referred to as the "zero-slope flow path algorithm". A MERIT DEM for Beaver Creek, Kentucky is used as a test case.

I. INTRODUCTION

The problem of computing topographic slope from a raster DEM (Digital Elevation Model) has been well-studied. Evans (1972) proposed a method based on fitting a quadratic function (degree 2 polynomial) of x and y to the 9 elevations in the vicinity of a grid cell. Zevenbergen and Thorne (1987) proposed a method based on fitting a partial quartic (degree 4 polynomial) function of x and y , again using the elevation of a grid cell and its 8 neighbors. Since then, there have also been papers comparing the relative merits of these two well-known methods, with the latter method appearing to be more susceptible to DEM inaccuracies or noise. Methods for computing slope, aspect, and various types of curvature (plan, profile, and streamline) are described in Peckham (2011). However, the problem of obtaining accurate estimates of along-channel slope from a DEM has received less attention and turns out to be very important for spatial hydrologic models that are based on DEMs. The purpose of this paper is to first review the essence of the problem and then to describe and compare two different algorithms for addressing this problem. It will be seen that the problem persists even when using DEMs with high vertical and horizontal resolution.

II. DESCRIPTION OF THE PROBLEM

Given a DEM with a vertical resolution of Δz and a horizontal resolution of Δx , the minimum, nonzero slope that is resolvable between a DEM grid cell and a nearest neighbor cell is: $\Delta z / \Delta x$. (It is slightly less than this between a grid cell and a diagonal neighbor.) So, for example, if Δz equals one meter and Δx equals 10 meters, slopes less than 0.1 will be unresolvable. This is a real problem because along-channel slopes in real rivers are often much smaller, typically between 10^{-4} and 10^{-6} . When channel slopes are too small to be resolved, one finds that along a channel flowline, the same elevation value occurs repeatedly many times (resulting in a slope of zero) before a lower elevation cell finally occurs. This can also result from filling pits in order to create a hydrologically sound DEM. Moving downstream, slope decreases and the number of repetitions before a drop in elevation tends to increase. At first glance, it may seem that increased vertical and horizontal resolution would resolve this issue. But decreasing Δz and Δx by the same factor does not change the minimum resolvable slope. Even if we decrease Δz to 0.01 meters (1 centimeter) but keep Δx to be 10 meters, slopes less than 0.001 will still be unresolvable and mapped to zero. Therefore, this problem is not addressed by using DEMs with higher vertical and horizontal resolution. Note that a slope of zero is not just wrong by some constant factor, it is completely unphysical, and even nonzero slopes can be off by factors of 100 to 1000 or more.

In a spatial hydrologic model, channel slope is an important variable because it determines the acceleration of the flow of water due to gravity. As a result, slope appears in Manning's formula for the flow velocity, v , given by:

$$v = (1/n) R_h^{2/3} S^{1/2} \quad (1)$$

where S is the along-channel slope, R_h is the hydraulic radius, and n is Manning's bed roughness parameter. The flow velocity is one of the most important variables in a hydrologic model and is needed to compute the volumetric discharge. The widely used kinematic wave approximation relies on this formula. But notice that if a grid cell has a slope of zero, this gives a flow velocity of zero, resulting in a tower of water above that grid cell. If the slope is wrong by a factor of 100, then the computed velocity for that grid cell is wrong by a factor of 10.

III. THE SLOPE-FROM-AREA ALGORITHM

Peckham (2011) introduced an algorithm to address this problem that is based on an empirical law known as Flint's Law. Flint's Law (Flint, 1974) is an observed relationship given by:

$$S = c A^\theta \quad (2)$$

that expresses the along-channel slope, S , at a basin's outlet as a power-law function of the basin's total contributing area, A . The exponent, θ , is negative (since slope decreases as area increases) and is between -1 and 0. It is sometimes called the concavity and is often close to -0.55 (Whipple, 2004).

To apply this algorithm, one first computes a grid of D8 flow directions and uses it to identify the grid cells that lie on the streamline of the main channel of the basin of interest. The main channel can be identified using a grid of total contributing area (computed by the D8 method) and repeatedly stepping upstream toward the D8 neighbor cell with the largest contributing area until a drainage divide is reached. According to Flint's Law, the predicted elevation for the k^{th} grid cell on the main channel is given by:

$$z_k(c, \theta) = z_0 + c \sum_{j=1, k} A_j^\theta \Delta L_j \quad (3)$$

where z_0 is the elevation of the outlet grid cell, A_j is the total contributing area of the j^{th} grid cell on the main channel, and ΔL_j is the horizontal distance between adjacent main-channel grid cells. A nonlinear, least-squares regression is then used to estimate the parameters c and θ in equation (3) that give the best fit to the main channel elevation values extracted from the DEM. Next, we assume that the same parameters c and θ are approximately valid for every other elevation profile in the DEM. This allows us to compute a new grid of channel slopes from the values in the grid of contributing areas. This grid of channel slopes is guaranteed to decrease monotonically downstream since contributing areas computed by the D8 method always increase downstream. The final step is to create a new DEM from this new slope grid. This is done using an iterative procedure, starting with the grid cells that are furthest downstream and then computing and adding up the small, floating-point elevation changes that must be made to upstream neighbor cells in order to achieve the slope prescribed by the new slope grid. The iteration continues upstream until every grid cell has the prescribed slope.

While contributing areas computed using the D8 method will always be strictly increasing in the downstream direction, a plot of contributing area vs. distance downstream will exhibit numerous discontinuities or jumps (Figure 3), especially wherever a large tributary flows into the main channel. As a result, channel slopes computed using this algorithm will also exhibit discontinuities at the same locations, but slopes will be strictly decreasing in the downstream direction. These discontinuities are likely a real feature of river networks and not an artifact of the algorithm.

Note that Flint's Law is not expected to apply to the concave down portion of a longitudinal profile that always occurs near ridgetops where diffusive hillslope processes are known to dominate over fluvial processes. As a result, this algorithm tends to produce elevations near ridges that are considerably larger than those in the original DEM, and slopes that are considerably steeper. However, up to a point, the steeper slopes and narrower ridgelines it generates can be closer to the actual topography as resolved in higher-resolution DEMs. Over most of the new elevation profile (the lower portion), elevations differ from those in the original DEM by less than a few meters.

As already explained, slopes computed between adjacent grid cells in a DEM can be off by very large factors. However, contributing areas measured from a DEM have much smaller relative errors, as long as grid cell size is much smaller than the basin size. Near the basin outlet, errors in an area grid can't be wrong by more than a small percentage. So even if Flint's Law is a crude approximation, using it to compute channel slopes from contributing areas is likely to be more accurate than computing slope between adjacent grid cells directly. Notice that this same algorithm could be used with any other empirical relationship that allows channel slope to be computed from contributing area.

IV. THE ZERO-SLOPE FLOW PATH ALGORITHM

This innovative algorithm has the option to only change the channel slopes in a DEM that have a value of zero. The idea here is to first find all of the grid cells in the DEM that have (1) a nonzero D8 slope -- this means its elevation is higher than that of the cell just downstream (called its D8 parent), and (2) a D8 parent cell with a slope of zero. For each of these cells, follow the D8 channel profile downstream (stepping to D8 parents) until you reach a grid cell that has a lower elevation (the final cell). Excluding the initial and final cells on this flow path, all other cells have the same elevation, and all but the last one (before the final cell) have a D8 slope value of zero. (See Figure 1.) Compute the along-channel flow distance between the initial and final cells, which will typically be much larger than the distance between adjacent grid cells. Next, compute a slope value using this distance and the elevation drop between the initial and final cells. Finally, assign this same slope value to all of the grid cells that have a D8 slope of zero between the initial and final cell. It is actually reasonable to assign this new slope value to the initial cell and the

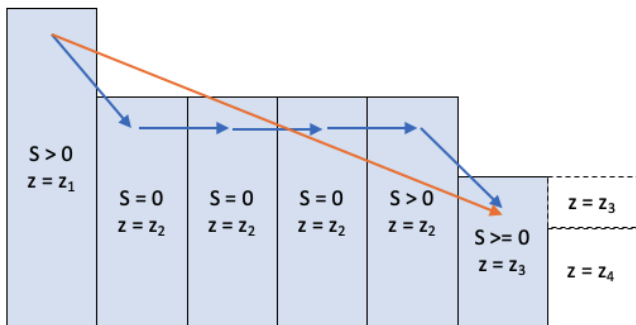


Figure 1. Diagram of a zero-slope flow path. The slope of the orange line is assigned to zero-slope cells and optionally to the first blue cell and the next-to-last blue cell, which have larger slopes.

next-to-last cell as well. This is optional but removes some unrealistic jumps in slope. Notice that it is possible for two (or more) such flow paths to merge, which leads to ambiguity as to which slope value to use downstream of their confluence. In this case, the lowest of the new slopes is used. Cells that initially had nonzero slopes are unaffected by this algorithm (unless the initial and next-to-last cell slopes are reassigned). If desired, a new DEM can be created from this new channel slope grid, as was done with the previous algorithm.

There are a few additional considerations when implementing this algorithm. One is that cells on the four edges of the DEM should be treated as having undefined (i.e., unreliable) D8 flow directions, and this can also be the case for other cells that flow to the edges. The algorithm does not assign slopes to these cells. Another issue is that there can be cells in flat areas that have a D8 slope of zero but no D8 children (upstream). Slopes can be assigned to these cells by allowing them to be the initial cell in a zero-slope flow path and allowing their slope to be changed. These cells have a contributing area of one grid cell and a slope of zero. This situation can be visualized by removing the first blue cell in Figure 1. Finally, notice that new slopes are assigned to short zero-slope paths before longer zero-slope paths. If saved to the same slope grid that is being used to follow paths, these can interfere with the proper assignment of slopes in the longer zero-slope paths. The solution is to use the original D8 slope grid for path-following and whenever a zero-slope cell lies on overlapping flow paths, smaller slope values overwrite larger ones.

V. COMPARISON OF THE TWO METHODS

A Jupyter notebook was created in order to analyze and compare these two algorithms and is available from the author. The figures in this section were created using a float-valued MERIT DEM for Beaver Creek, Kentucky, with a horizontal resolution of 3 arcseconds. Figure 2 shows the longitudinal profile

of the main channel, extracted from the DEM, and the smooth profile computed using the slope-from-area algorithm. The best-fit Flint's Law parameters are $c=0.1191$ and $\theta=-0.886$. Figure 4 shows the log of the slopes of the profile computed by this algorithm. The jumps in this plot correspond to jumps in the D8 areas for the main channel, caused by larger channels that flow into the main channel (Fig. 3). Figure 5 shows the profile of D8 slopes as measured from the DEM (many of which are zero), as well as the slopes computed from the slope-from-area algorithm. Figure 6 shows the profile of slopes computed using the slope-from-area algorithm, as well as those computed using the zero-slope flow path algorithm. While there are still jumps, the zero-slope flow path algorithm performs much better than the simple D8 slope algorithm. Further refinements to the algorithm are likely possible to eliminate some of these remaining jumps in slope.

VI. CONCLUSIONS

We have described and compared two algorithms for more accurately computing along-channel slopes from DEMs. This problem is distinct from the problem of computing slopes in more upland regions. The slope-from-area algorithm was previously described by Peckham (2011) while the zero-slope flow path algorithm is new. Both algorithms eliminate zero-slope grid cells and offer dramatic improvements over the simple D8 slope algorithm. Both have been implemented in a spatially distributed hydrologic model written in Python called TopoFlow (Peckham et al., 2017) and provide good results in this context. It is worth noting that the slope-from-area algorithm is based on an idealization that may only apply in more mature landscapes and it produces slopes that are everywhere significantly different from those in the original DEM. It also produces slopes and elevations that are too large if used near ridgelines. Except in the steepest parts of the basin, it produces new elevations that, while different, are within a few meters of the original values. The zero-slope flow path algorithm may be preferable in many situations since it preserves the original elevations and slopes except where they are

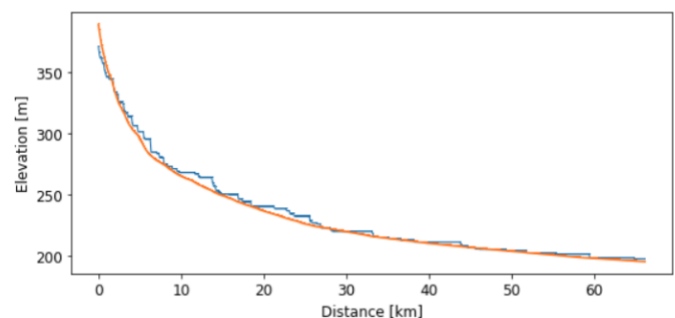


Figure 2. Main channel elevations of the Beaver Creek DEM (blue) and those from the slope-from-area algorithm. Best-fit Flint's Law parameters: $c=0.1191$ and $\theta=-0.886$. The uppermost portion of the profile has been excluded.

problematic. While the zero-slope flow path algorithm only applies if there are zero-slope grid cells in the DEM, this can occur in several situations such as (1) after filling depressions in DEMs that aren't hydrologically sound, (2) after applying a flow-enforcement or "stream-burning" algorithm, (3) when using older, integer-valued DEMs, or (4) when using floating-point DEMs when only 1 to 3 digits after the decimal are considered significant or meaningful. By contrast, the slope-from-area algorithm can be applied to DEMs with no zero-slope cells. Note that D8 slopes in channels can still be inaccurate, even for hydrologically sound, float-valued DEMs, for the reasons explained in the Introduction. Moreover, algorithms to enforce hydrologic soundness can also introduce inaccuracies. For this short paper, a non-hydrologically-sound, float-valued MERIT DEM was used. Further refinements of these algorithms will be explored in future work.

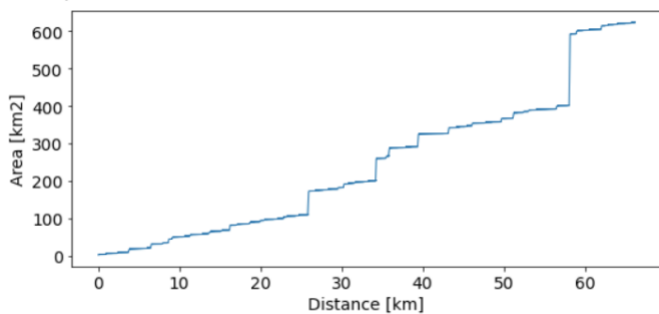


Figure 3. D8 total contributing area on main channel of Beaver Creek DEM, with jumps at major confluences.

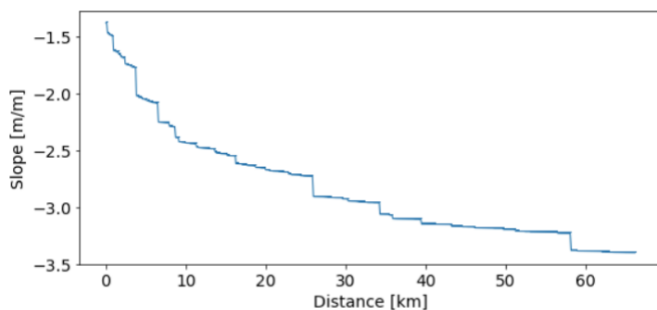


Figure 4. Base 10 log of slopes on main channel of Beaver Creek DEM computed with the slope-from-area algorithm.

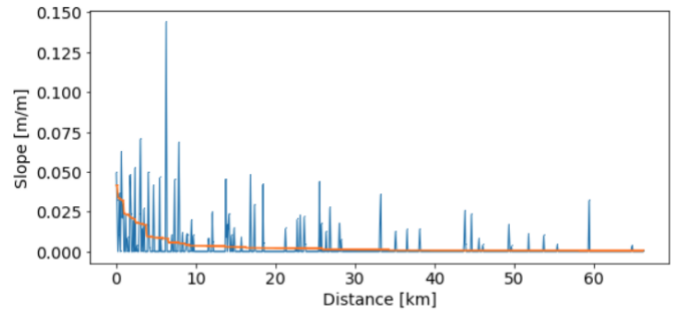


Figure 5. D8 slope on main channel of Beaver Creek DEM (blue) and slope from slope-from-area algorithm.

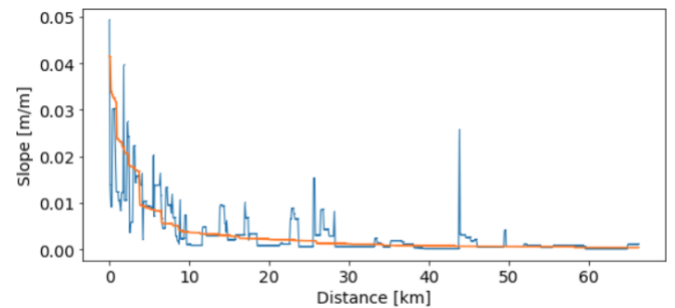


Figure 6. Slopes on main channel of Beaver Creek DEM computed using both the slope-from-area algorithm (orange) and the zero-slope flow path algorithm (blue).

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